

# Noise Calculator, How Do You Use it in Design?

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## *One of Many Definitions*

noise - electrical or acoustic activity that can disturb communication



 TEXAS  
INSTRUMENTS



## Another Definition

**NOISE** - the quality of lacking  
any predictable order or plan





## Noise Categories

1. Extrinsic Noise—
  - 50 / 60 Hz Line Noise...RFI / EMI
  - Charge displacement (very hi-Z circuit)
  - Switching power supplies...Micro-phonic
  - Digital noise...Ground loops
  - Atmospheric...Cosmic
2. Intrinsic Noise—
  - Thermal Noise
  - Shot Noise
  - 1/f Noise
  - Popcorn (Burst) Noise



## Thermal Noise

$$e^2 = 4kTR\Delta f$$

Where:

- k = Temperature (°K)
- R = Resistance ( $\Omega$ )
- f = frequency (Hz)
- k = Boltzmann's constant  
( $1.38\text{E-}23$  joule/°K)
- e = volts ( $V_{\text{RMS}}$ )



Also known as “Johnson Noise”— first observed by Schottky in 1918,  
first measured by Johnson in 1928 & soon after formalized by Nyquist.

Thermal noise is produced by random motion of charges.

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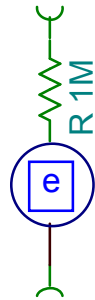
Thermal noise is produced by random motion of charges.

$e = \sqrt{4kTR\Delta f}$  (when  $\Delta f = 1\text{Hz}$ , "e" is the voltage noise spectral density)



## Resistor Thermal Noise

Thevenin  
Equivalent



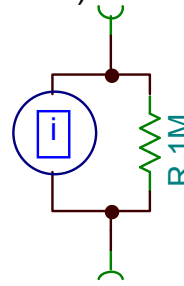
$$e = \sqrt{4kTR\Delta f}$$

$$e = 128 \text{ nV}/\sqrt{\text{Hz}}$$

Voltage noise  
spectral density

At:  $T = 298 \text{ °K}$  ( $25 \text{ °C}$ )

Norton  
Equivalent



$$i = \sqrt{4kT\Delta f/R}$$

$$i = 128 \text{ fA}/\sqrt{\text{Hz}}$$

Current noise  
spectral density



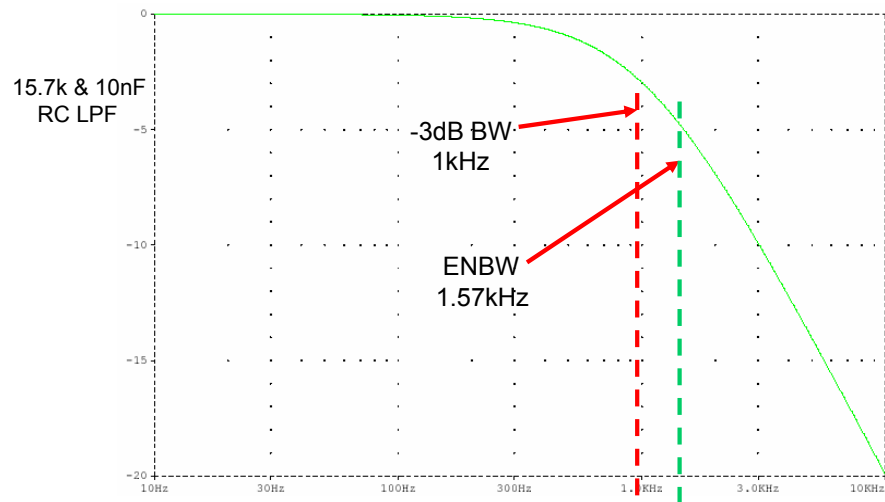
By simply using Ohm's Law we can obtain the mean- square short- circuit noise current from

$$i^2 = e^2/R^2 = 4kT\Delta f/R$$

$$i = \sqrt{4kT\Delta f/R} \quad (\text{when } \Delta f = 1\text{Hz, "i" is the current noise spectral density})$$



## Low- Pass ENBW Example



Correction factor for 1- pole filter = 1.57, 1kHz (-3dB BW) \* 1.57 = 1.57kHz ENBW



### 16.2.5 Equivalent Noise Bandwidth

Equivalent noise bandwidth is the ratio of the input noise power to the noise power in the output of an FFT filter times the input data sampling rate. Every signal contains some noise. That noise is generally spread over the frequency spectrum of interest, and each narrowband filter passes a certain amount of that noise through its main lobe and sidelobes. White noise is used as the input signal and the noise power out of each filter is compared to the noise power into the filter to determine the equivalent noise bandwidth of each passband filter. In other words, equivalent noise bandwidth represents how much noise would come through the filter if it had an absolutely flat passband gain and no sidelobes. It should be noted that leakage to the side lobes not only decreases the magnitude of the main lobe but also increases the frequency resolution (spreading) of the main lobe. The width of the spread is compared to the rectangular window, which has a main lobe width of 1. Table 16.1 shows the equivalent bandwidth spread to be greater than 1 for all window functions.

[http://books.google.com/books?id=QxJivJTJLy8C&pg=PA183&pg=PA183&dq=Filter+Equivalent+Noise&source=web&ots=B6C0imt-iE&sig=UKhOqs3jhnDINpLKfKzKak0eDGs&hl=en&sa=X&oi=book\\_result&resnum=6&ct=result#PPA183,M1](http://books.google.com/books?id=QxJivJTJLy8C&pg=PA183&pg=PA183&dq=Filter+Equivalent+Noise&source=web&ots=B6C0imt-iE&sig=UKhOqs3jhnDINpLKfKzKak0eDGs&hl=en&sa=X&oi=book_result&resnum=6&ct=result#PPA183,M1)







## Filter Equivalent Noise BW

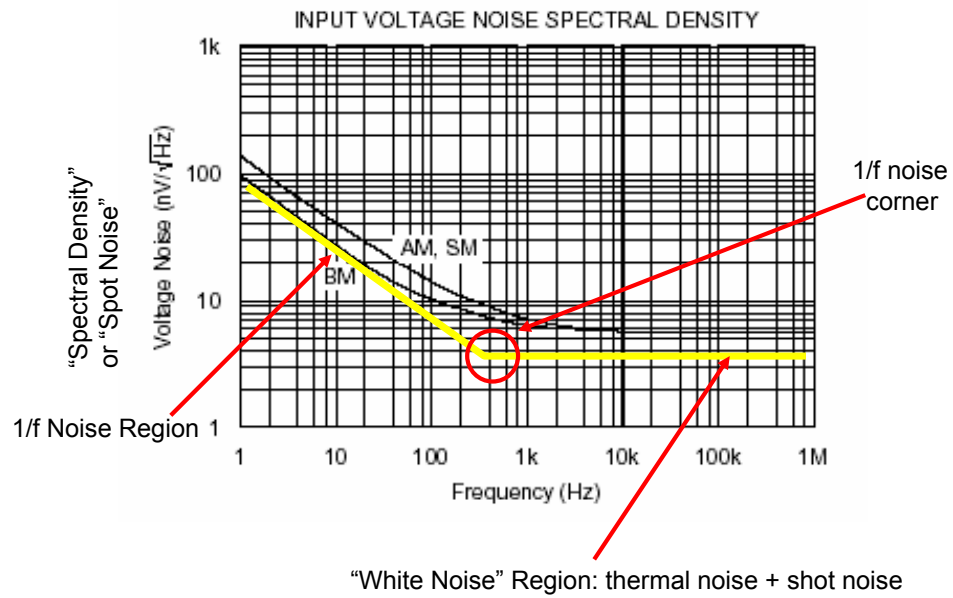
Butterworth (fco = 3 dB)		Chebyshev (fco = ripple)						Bessel (fco = 3 dB)	
Order	EqNBW	Ripple	0.01 dB	0.1 dB	0.25 dB	0.5 dB	1.0 dB	Order	EqNBW
1	1.5708	Order						1	1.57
2	1.1107	2	3.667 2	2.14 44	1.744 9	1.488 9	1.253 2	2	1.56
3	1.0472	3	1.964 2	1.44 18	1.282 5	1.166 6	1.041 1	3	1.08
4	1.0262	4	1.503 9	1.23 26	1.140 5	1.065 6	0.973 5	4	1.04
5	1.0166	5	1.311 4	1.14 17	1.078 0	1.020 8	0.943 3	5	1.04
6	1.0115	6	1.212 0	1.09 37	1.044 8	0.997 0	0.927 2	6	1.04
7	1.0084	7	1.153 7	1.06 53	1.025 1	0.982 8	0.917 5		
8	1.0065	8	1.116 6	1.04 71	1.012 5	0.973 6	0.911 33		
9	1.0051	9	1.091 4	1.03 47	1.003 8	0.967 4	0.907 1		
10	1.0041	10	1.073 6	1.02 58	0.997 7	0.962 9	0.904 1		

Reference: [http://www.rfcafe.com/references/electrical/filter\\_eqnbw.htm](http://www.rfcafe.com/references/electrical/filter_eqnbw.htm)





## Amplifier Noise Spectrum





## Noise Temporal Behavior

$$P(f) = 1 / f^{\text{beta}}$$



1/f noise spectrum when **beta = 1**, equal power per octave

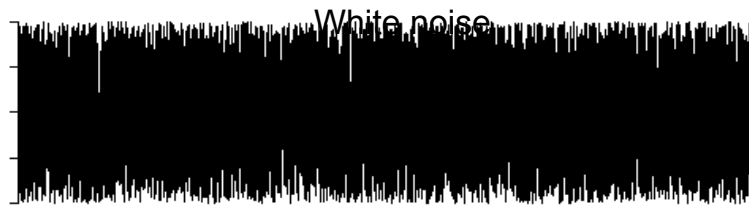


<http://astronomy.swin.edu.au/~pbourke/fractals/noise/>





## Noise Temporal Behavior



2-pole LPF (integrate) white noise and  
you get Brownian noise, **beta = 2**



<http://astronomy.swin.edu.au/~pbourke/fractals/noise>





## Noise Temporal Behavior

beta = 1  
1/f noise  
("flicker noise")



beta = 2  
Brownian noise  
("random walk")



beta = 3



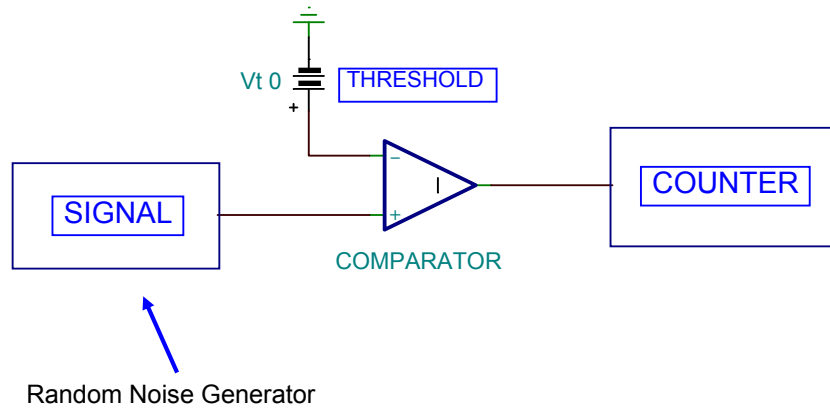
<http://astronomy.swin.edu.au/~pbourke/fractals/noise/>





## A Noise Experiment

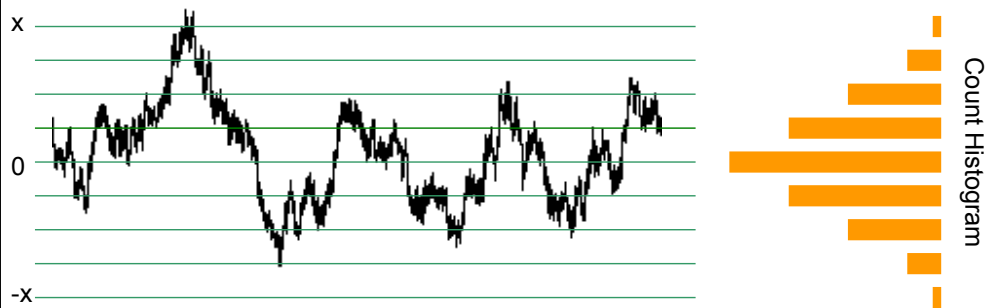
Vary  $V_t$  from  $x$  to  $-x$  & record counter reading





## Finding the Amplitude Distribution

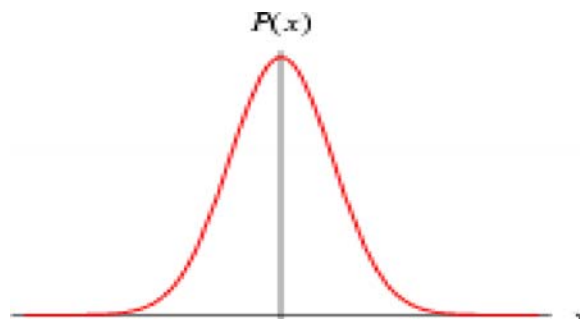
A counter records the input signal amplitude as a function of “x”– the comparator threshold.



Take a large number of samples “N”



## Distribution Curve Terms



**Math majors: “Normal Distribution”**

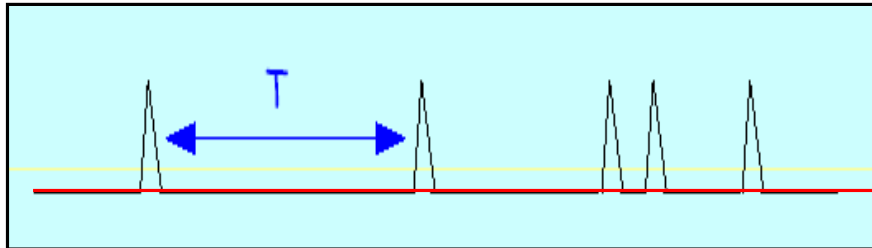
**Physics majors: “Gaussian Distribution”**

**Sociology Majors: “Bell Curve”**





## Shot Noise



Shot noise is a series of quantized charge packets (pulses of fixed amplitude) with a completely random time ( $T$ ) of arrival. This figure represents an extremely low current where individual electrons can be seen. Although not common, this type of noise is encountered in photon-counting detectors using a cooled photomultiplier tube (PMT).

Note that the quantized nature of shot noise has completely different probability statistics than Gaussian noise. Shot noise obeys Poisson statistics, although at higher current it approaches Gaussian.

The mean (red) power of shot noise depends on its amplitude ( $nq/T$ )



## Shot Noise

$$i^2 = 2qI\Delta f$$

Where:  $I$  is the current in amps (A)

$q$  is the electronic charge,  $1.6E-19$  c

The current noise spectral density of shot noise is:  $i = \sqrt{2qI}$

*shot noise is not Gaussian.*

Reference: <http://homepages.cae.wisc.edu/~gubner/fpphist.shtml>



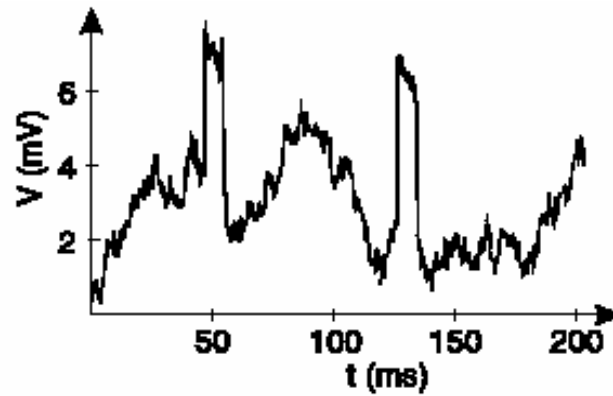
Shot noise is generated by the random emission of electrons (such as from a cathode or photocathode) or by the random passage of charge carriers (electrons and holes) across a potential barrier. It was first reported by Campbell in 1909.

The mean-square shot noise current in the frequency band  $\Delta f$  is given by an equation developed by Schottky between 1918 to 1928:

Although shot noise is a type of white noise, it does not have the same statistical distribution as thermal noise—



## Burst Noise Characteristics



Burst noise is primarily a current noise.

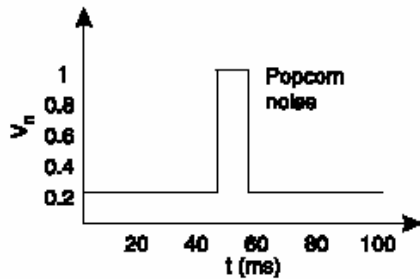
Reference: [http://www.wat.edu.pl/review/optor/11\(1\)45.pdf](http://www.wat.edu.pl/review/optor/11(1)45.pdf)



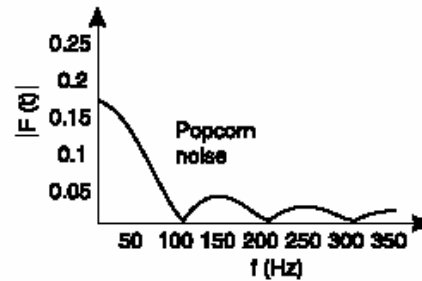
Burst noise (also known as “Popcorn Noise”) is thought to be caused by lattice defects but it is not completely understood. Microscale beta shifts are also suspected. This type of noise is usually associated with bipolar transistors but it is also seen in pyroelectric detectors.



## Burst Noise Characteristics



*Time  
Domain*



*Frequency  
Domain*

Reference: [http://www.wat.edu.pl/review/optor/11\(1\)45.pdf](http://www.wat.edu.pl/review/optor/11(1)45.pdf)



A representation of a typical noise burst is shown on the left as it would appear on an oscilloscope. Performing an FFT on this waveform yields the spectrum shown on the right. Note that the majority of the pulse energy is concentrated at low frequencies but a portion of it is spread over much higher frequencies.



## From the Data Sheet

PARAMETER	CONDITION	OPA277P, U OPA2277P, U			UNITS
		MIN	TYP <sup>(1)</sup>	MAX	
<b>NOISE</b>					
Input Voltage Noise, $f = 0.1$ to $10\text{Hz}$			0.22		$\mu\text{Vp-p}$
Input Voltage Noise Density, $f = 10\text{Hz}$ $e_n$			0.035		$\mu\text{Vrms}$
$f = 100\text{Hz}$			12		$\text{nV}/\sqrt{\text{Hz}}$
$f = 1\text{kHz}$			8		$\text{nV}/\sqrt{\text{Hz}}$
$f = 10\text{kHz}$			8		$\text{nV}/\sqrt{\text{Hz}}$
Current Noise Density, $f = 1\text{kHz}$ $i_n$			0.2		$\text{pA}/\sqrt{\text{Hz}}$

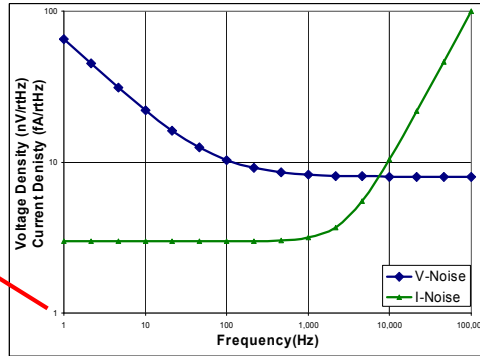
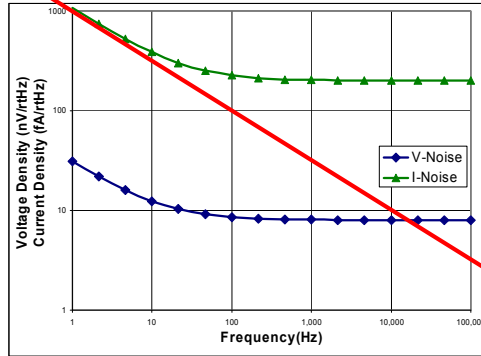
PARAMETER	CONDITION	OPA132P, U OPA2132P, U			UNITS
		MIN	TYP	MAX	
<b>NOISE</b>					
Input Voltage Noise					
Noise Density, $f = 10\text{Hz}$			23		$\text{nV}/\sqrt{\text{Hz}}$
$f = 100\text{Hz}$			10		$\text{nV}/\sqrt{\text{Hz}}$
$f = 1\text{kHz}$			8		$\text{nV}/\sqrt{\text{Hz}}$
$f = 10\text{kHz}$			8		$\text{nV}/\sqrt{\text{Hz}}$
Current Noise Density, $f = 1\text{kHz}$			3		$\text{fA}/\sqrt{\text{Hz}}$





## Typical Curves

**1/f**



**OPA277 -Bipolar**

**OPA132 -FET**

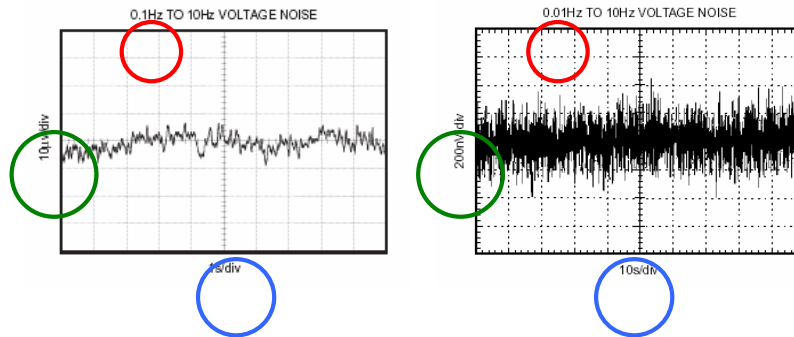




## Data Sheet Noise Info

Op amp and amplifier data sheets may specify Voltage noise two ways—

1. Input voltage noise density @ a spec'd frequency in  $\text{nV}/\sqrt{\text{Hz}}$ .
2. Voltage noise within a frequency band of 0.1Hz to 10Hz in  $\mu\text{V}_{\text{P-P}}$
3. Graphical low- frequency noise plots such as these:



Choose one...





## Source Resistance Noise

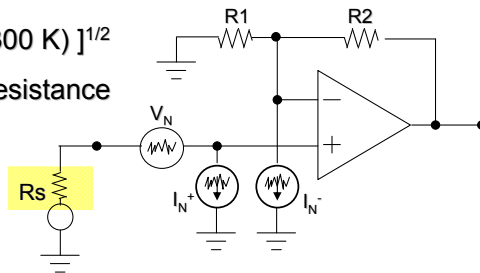
### Noise from source resistance:

$$V_n = (4 R k T B)^{1/2} \quad (\text{total noise Bandwidth})$$

$$V_n = (4 R k T)^{1/2} \quad (\text{noise density})$$

e.g. for 10k $\Omega$  source:

$$V_n = [ 4 (10,000\Omega) (1.38 \times 10^{-23}) (300 \text{ K}) ]^{1/2}$$
$$= 12.8 \text{ nV}/\sqrt{\text{Hz}} \text{ from source resistance}$$

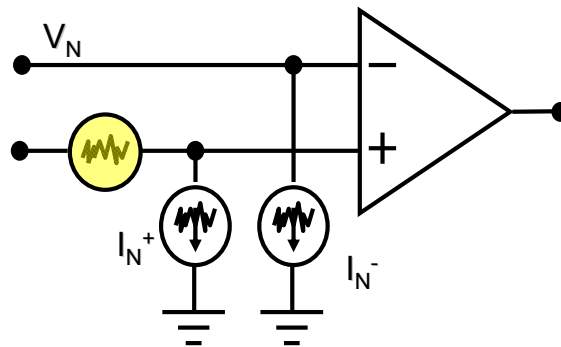






## Voltage Noise

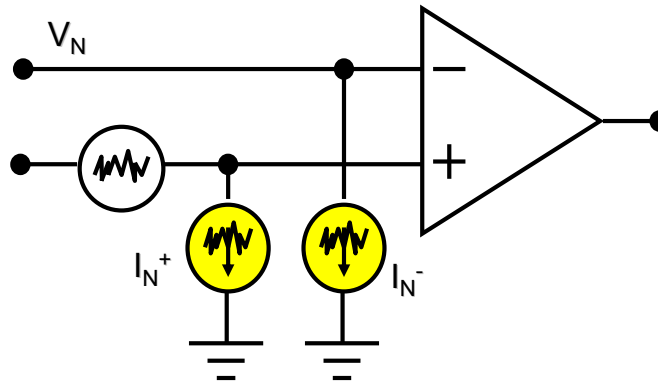
A time-varying component of offset voltage.





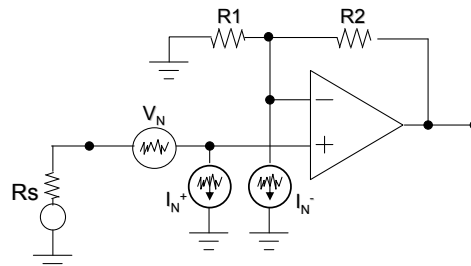
## Current Noise

A time-varying component of input bias current.





## The Noise Model



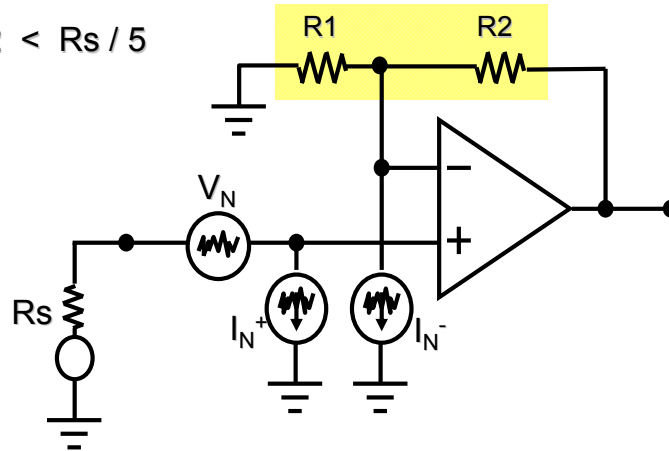
- $R_s$  : Thermal noise  $\Rightarrow$  to RTI noise
- $V_N$  : Op amp  $V_N \Rightarrow$  RTI noise
- $I_N^+ : (I_N^+) (R_s) \Rightarrow$  RTI noise
- $I_N^- : (I_N^-) (R_1 \parallel R_2) \Rightarrow$  RTI noise
- $R_1$  : (Thermal noise)  $(G-1)/G \Rightarrow$  RTI noise
- $R_2$  : (Thermal noise)/ $G \Rightarrow$  RTI noise



## Feedback Resistors

- Add thermal noise
- React with op amp's current noise

Make:  $R1 \parallel R2 < R_s / 5$





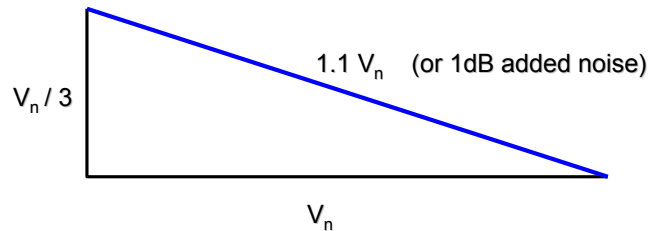
Noise “adds” as right-angle vectors

Or

Root Sum Square (RSS)

NOT .....RMS

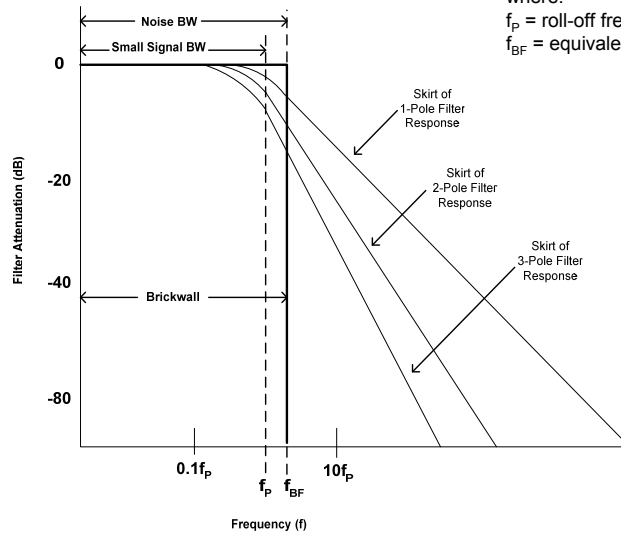
$$V_{nt} = \sqrt{V_{n1}^2 + V_{n2}^2} = \sqrt{V_{n1}^2 + \left(\frac{V_{n1}}{3}\right)^2}$$



When a noise source is one-third or less than the dominate source, it is virtually insignificant.



## Real Filter Correction vs Brickwall Filter



where:  
 $f_P$  = roll-off frequency of pole or poles  
 $f_{BF}$  = equivalent brickwall filter frequency



## Noise Bandwidth Ratios for n<sup>th</sup> Order Low-Pass Filters

$BW_n = (f_H)(K_n)$  Effective Noise Bandwidth

### Real Filter Correction vs Brickwall Filter

Number of Poles in Filter	$K_n$ Noise Bandwidth Ratio
1	1.57
2	1.22
3	1.16
4	1.13
5	1.12



## STDEV Relationship to Peak-to-Peak

Number of Standard Deviations	Percent chance of measuring voltage
$2\sigma$ (same as $\pm\sigma$ )	68.3%
$3\sigma$ (same as $\pm 1.5\sigma$ )	86.6%
$4\sigma$ (same as $\pm 2\sigma$ )	95.4%
$5\sigma$ (same as $\pm 2.5\sigma$ )	98.8%
$6\sigma$ (same as $\pm 3\sigma$ )	99.7%

**Is standard deviation the same as RMS?**







## Calculating Noise Vpp from Noise Vrms

### Relation of Peak-to-Peak Value of Noise Voltage to rms Value

Peak-to-Peak Amplitude	Crest Factor (CF)	Probability of Having a Larger Amplitude
2 X rms	1	32%
3 X rms	1.5	13%
4 X rms	2	4.6%
5 X rms	2.5	1.2%
6 X rms *	3	0.3%
7 X rms	3.5	0.05%

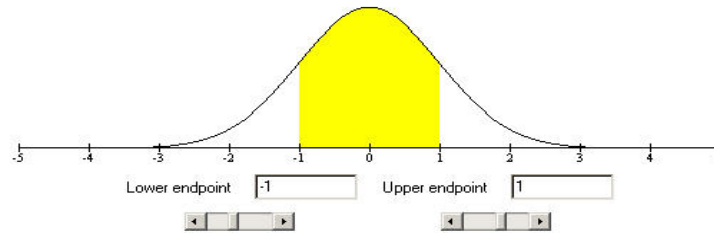
**\*Common Practice is to use:  
Peak-to-Peak Amplitude = 6 X rms**





# Standard Deviation $\sigma$

Standard Normal Curve



Highlighted area: 68.3%

Probability that instantaneous  
value is **less** than RMS value

$1\sigma = 68.3\%$

$2\sigma = 95.5\%$

$3\sigma = 99.73\%$

$4\sigma = 99.9937\%$

Probability that instantaneous  
value is **greater** than RMS value

$1\sigma = 31.7\%$

$2\sigma = 4.5\%$

$3\sigma = 0.27\%$

$4\sigma = 0.0063\%$

Reference: <http://stat-www.berkeley.edu/~stark/Java/NormHiLite.htm>



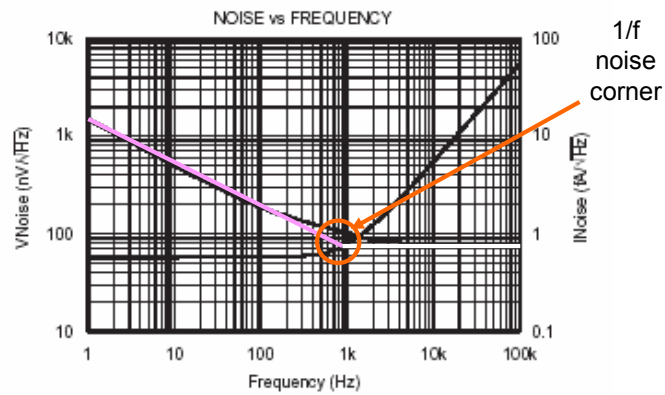


## The Limit to LP Filtering

CMOS INA321  
voltage noise  
spectral density

White: white  
noise region

Pink:  $1/f$  (pink)  
noise region



LP filtering in this region  
produces no improve-  
ment in noise. Why?

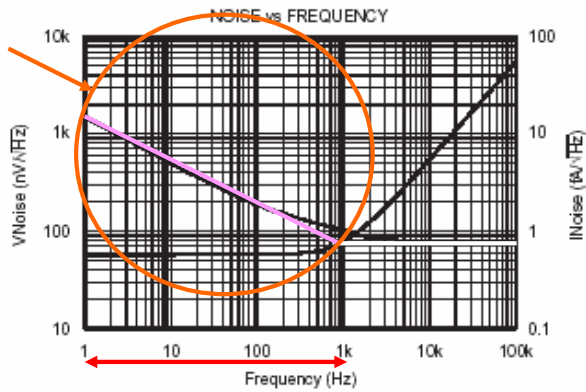
LP filtering in this region  
reduces noise by the  
square- root of BW.





## The Dreaded 1/f Region

LP filtering in this region produces no improvement in noise. Why?



Because in the 1/f noise region the noise density increases at the same rate that you reduce the bandwidth! In the INA321 example above look at what reducing the BW by a factor of 100 (from 1kHz to 10Hz) does. White noise would be reduced by a factor of  $\sqrt{100}$  or 10 but in the 1/f region the noise increased by a factor of 10 as the  $\sqrt{BW}$  was reduced by 10— no improvement!